

Bi-Large neutrino mixing with charged lepton correction.

Subhankar Roy* and N. Nimai Singh†

Department of Physics, Gauhati University, Guwahati-781014, India

December 3, 2012

Abstract

The usual Bi-Maximal (BM) neutrino mixing faces an inherent problem in lowering the solar angle below $\tan^2 \theta_{12} = 0.50$ when charged lepton correction is taken. This minimum θ_{12} is achievable only if CP violation is absent. We start with a new model which incorporates a new idea of mixing developed recently, called Bi-Large (BL) mixing, similar to BM mixing except that the former chooses rather θ_{13} as Cabibbo angle (θ_c) than zero. We apply this mixing in the neutrino sector followed by a charged lepton correction with the CKM type matrix U_l . The model marks a prediction on θ_{23} to lie within the first octant. The CP violating phase δ_{CP} dictates the prediction of all the three mixing angles. A proper choice of δ_{CP} , leads to the predictions of all the three mixing angles including θ_{12} , to align very precisely with the experimental bestfit. This close agreement thus hoists Bi-Large mixing as an important and promising mixing scheme, in contrast to BM or TBM mixing as a first approximation. A formal derivation of BL mixing from discrete symmetry will be an important investigation in neutrino physics.

PACS numbers: 14.60.Pq, 12.15.Ff.

Keywords: Bi-Large mixing, Bi-maximal mixing, Tri-bimaximal mixing, neutrino masses, Cabibbo angle, CKM matrix, PMNS Matrix .

1 Introduction

The recent experimental data from Double Chooz [1], Daya Bay [2], RENO [3], T2K [4] and MINOS [5] collaborations, indicate not only a nonzero reactor angle (θ_{13}) but also with its magnitude of the order of Cabibbo angle (θ_c). Tri-Bimaximal (TBM) mixing [6] and Bimaximal (BM) mixing [7, 8, 9] are two popular mixing patterns which predict $\sin \theta_{13} = 0$. TBM mixing has a strong theoretical support because of its relation with A_4 [10 - 14], one of the candidates of discrete flavour symmetry groups. From theoretical point of view, small deviation of the order of square of λ_c (where $\lambda_c = \sin \theta_c \approx 0.22$) is expected. But a large correction of the order of λ_c to $\sin \theta_{13} = 0$, clearly interrogates the loyalty of TBM mixing as a first approximation. This was pointed out in the literature [15]. The same argument holds good for BM mixing scheme also. In addition, at the Neutrino 2012 conference the MINOS collaboration hinted for a non-maximal θ_{23} , which also goes against the TBM and BM predictions. From the analyses given in Ref. [16, 17], θ_{23} tilts towards a preference for $\theta_{23} < 45^\circ$.

A new idea of mixing scheme called Bi-Large (BL) mixing [15] has been proposed recently by Boucenna *et.al*, apart from the existing TBM and BM mixing schemes. They considered $\sin \theta_{13}$ as the fundamental parameter (λ) and the idea behind this ansatz lies in the smallness of θ_{13} , among the three mixing parameters. They expressed $\sin \theta_{12}$ and $\sin \theta_{23}$ as linear functions of λ . Thus,

$$\sin \theta_{13} = \lambda, \quad \sin \theta_{12} = a\lambda, \quad \sin \theta_{23} = s\lambda. \quad (1)$$

Here a, s are free parameters and $a \simeq s$. The resulting parametrization neither terminates to TBM nor BM pattern as limiting cases, though maximal atmospheric angle can be obtained. When $\lambda \rightarrow 0$, the

*Email: meetsubhankar @ gmail.com

†E-mail: nimai03 @ yahoo.com

neutrinos are unmixed. From simple numerical analysis they have shown that strict BL mixing occurs when $\lambda \simeq \lambda_c \approx 0.22$ and under that condition we get $a = s = 3$.

We start with this strict BL ansatz [Eq.(1)] where the Cabibbo angle (λ_c), the most important parameter from CKM matrix generates the whole parametrization in the neutrino sector. We take,

$$\sin \theta_{13} = \lambda_c, \quad \sin \theta_{12} = 3\lambda_c, \quad \sin \theta_{23} = 3\lambda_c \quad . \quad (2)$$

Pending a formal derivation of the BL mixing from a discrete symmetry, we wish to explore its matrix form from phenomenological ground. Following the standard PDG scheme of parametrization, we arrive at the following strict BL mixing matrix (U_{BL}),

$$U_{BL} = \begin{pmatrix} \frac{3}{4}(1 - \frac{\lambda_c^2}{2}) & \frac{\sqrt{7}}{4}(1 - \frac{\lambda_c^2}{2}) & \lambda_c \\ -\frac{3\sqrt{7}}{16}(1 + \lambda_c) & \frac{9}{16}(1 - \frac{7}{9}\lambda_c) & \frac{\sqrt{7}}{4}(1 - \frac{\lambda_c^2}{2}) \\ \frac{7}{16}(1 - \frac{9}{7}\lambda_c) & -\frac{3\sqrt{7}}{16}(1 + \lambda_c) & \frac{3}{4}(1 - \frac{\lambda_c^2}{2}) \end{pmatrix}, \quad (3)$$

$$= \begin{pmatrix} 0.7309 & 0.6446 & 0.2257 \\ -0.608 & 0.4637 & 0.6446 \\ 0.3105 & -0.608 & 0.7309 \end{pmatrix}.$$

U_{BL} satisfies unitarity condition. If we first approximate the neutrino mixing matrix U_ν to U_{BL} and then in order to account for the required deviations, we consider the correction from charged lepton sector [18]. We try to find out the possible texture of charged lepton matrix U_l (must follow unitarity condition), which may serve our purpose.

2 The problems in Bi-maximal (BM) mixing

The strict BL mixing [15] and BM mixing patterns have certain similarities. θ_{12} and θ_{23} are equal for both the cases. The former predicts them to be 41° and the later takes them as maximal i.e., 45° . The significant difference lies in the fact that the former starts with $\theta_{13} = \theta_c$, and later with $\theta_{13} = 0^\circ$,

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

In the reference [18], the authors put forward an viable technique to comply with the experimental data. This is summarised as follows. In fact they considered $U_\nu = U_{BM}$ and then performed a charged lepton correction by choosing the charged lepton matrix U_l to be CKM type,

$$U_l = \begin{pmatrix} 1 - \frac{\lambda_c^2}{2} & \lambda_c e^{i\delta_{cp}} & 0 \\ -\lambda_c e^{-i\delta_{cp}} & 1 - \frac{\lambda_c^2}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (5)$$

The possible inclusion of Dirac phase δ_{cp} in 1-2 and 2-1 positions of U_l was first introduced by Fritzsch and Xing [19]. In eq (5), U_l satisfies unitarity condition. The $U_{PMNS} = U_l^\dagger U_\nu$ becomes,

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}}(1 - \frac{\lambda_c^2}{2}) + \frac{\lambda_c}{2}e^{i\delta_{cp}} & \frac{1}{\sqrt{2}}(1 - \frac{\lambda_c^2}{2}) - \frac{\lambda_c}{2}e^{i\delta_{cp}} & -\frac{\lambda_c}{\sqrt{2}}e^{i\delta_{cp}} \\ \frac{1}{2}(\frac{\lambda_c^2}{2} - 1) + \frac{\lambda_c}{\sqrt{2}}e^{-i\delta_{cp}} & \frac{1}{2}(1 - \frac{\lambda_c^2}{2}) + \frac{\lambda_c}{\sqrt{2}}e^{-i\delta_{cp}} & \frac{1}{\sqrt{2}}(1 - \frac{\lambda_c^2}{2}) \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (6)$$

From Eq.(6), using the following relations,

$$\sin^2 \theta_{13} = |U_{e3}|^2, \quad \sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2}, \quad (7)$$

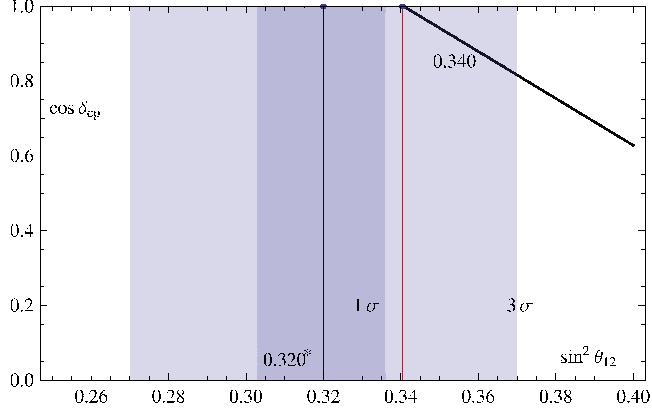


Figure 1: The dependence of $\cos \delta_{cp}$ on $\sin^2 \theta_{12}$ for BM case with charged lepton correction. The prediction of the solar angle can not be lowered to the present experimental best-fit through any possible way. The lowering of θ_{12} upto certain level is possible at the cost of $\delta_{cp} \rightarrow 0, 2\pi$.

we obtain

$$\sin^2 \theta_{13} = \frac{\lambda_c^2}{2} \approx 0.0254, \quad (8)$$

$$\sin^2 \theta_{12} = \frac{4 - 2\lambda_c^2 + 2\sqrt{2}\lambda_c(\lambda_c^2 - 2)\cos \delta_{cp}}{8(1 - \frac{\lambda_c^2}{2})}, \quad (9)$$

$$\sin^2 \theta_{23} = \frac{1}{2}(1 - \frac{\lambda_c^2}{2}) \approx 0.488, \quad (10)$$

$$J_{CP}^{BM} \approx \frac{1}{4\sqrt{2}}\lambda_c \sin \delta_{cp}. \quad (11)$$

The prediction of θ_{13} matches with the best-fit value [20], while that for θ_{23} lies within 2σ [20]. The prediction of θ_{12} depends on δ_{cp} . Now if we want $\sin^2 \theta_{12}$ as 0.32 (best-fit) [20], from Eq.(9), we have $\cos \delta_{cp} = 1.13$, which is absurd. The relation between $\sin^2 \theta_{12}$ and δ_{cp} is illustrated in Fig.1. The minimum value of 0.3407 for $\sin^2 \theta_{12}$ (i.e., $\tan^2 \theta_{12} \approx 0.52$) is obtained at the cost of $\cos \delta_{cp} = 1$, which in turn gives CP violation parameter Jarlskog invariant $J_{CP}^{BM} = 0$. This is the discrepancy of BM model where $\sin^2 \theta_{12}$ can not be suppressed even though J_{CP} is sacrificed.

3 Strict Bi-Large mixing and Charged lepton contribution

We now assume that neutrino mixing matrix U_ν follows strict BL mixing [Eq.(2), Eq.(3)] and take $U_\nu = U_{BL}$. We assume the charged lepton mixing matrix to be CKM type. Motivated by the similarities among the two mixing schemes and the partial success, we try with the same CKM type U_l employed for BM case (Eq.(5)) [18] and generate $U_{PMNS} = U_l^\dagger U_{BL}$.

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad (12)$$

where,

$$\begin{aligned} U_{e1} &= \frac{3}{16}\{(\lambda_c^2 - 2)^2 + \sqrt{7}\lambda_c(1 + \lambda_c)e^{i\delta_{cp}}\}, \\ U_{e2} &= \frac{1}{16}\{\sqrt{7}(\lambda_c^2 - 2)^2 + \lambda_c(7\lambda_c - 9)e^{i\delta_{cp}}\}, \\ U_{e3} &= \frac{1}{8}\lambda_c(\lambda_c^2 - 2)(\sqrt{7}e^{i\delta_{cp}} - 4), \\ U_{\mu 1} &= \frac{3}{32}(\lambda_c^2 - 2)\{\sqrt{7}(1 + \lambda_c) - 4\lambda_ce^{-i\delta_{cp}}\}, \end{aligned}$$

$$\begin{aligned}
U_{\mu 2} &= \frac{1}{32}(2 - \lambda_c^2)(9 - 7\lambda_c + 4\sqrt{7}e^{-i\delta_{cp}}), \\
U_{\mu 3} &= \frac{\sqrt{7}}{16}(\lambda_c^2 - 2)^2 + \lambda_c^2 e^{-i\delta_{cp}}, \\
U_{\tau 1} &= \frac{1}{16}(7 - 9\lambda_c), \\
U_{\tau 2} &= -\frac{3\sqrt{7}}{16}(1 + \lambda_c), \\
U_{\tau 3} &= \frac{3}{8}(2 - \lambda_c^2).
\end{aligned}$$

Following Eq. (7) and from Eq. (13), we get

$$\sin^2 \theta_{13} = \frac{1}{64}\lambda_c^2(\lambda_c^2 - 2)^2(23 - 8\sqrt{7}\cos \delta_{cp}), \quad (13)$$

$$\sin^2 \theta_{12} = \frac{112 + \lambda_c^2\{7\lambda_c(31\lambda_c - 18) - 143\} + 2\sqrt{7}\lambda_c(7\lambda_c - 9)(\lambda_c^2 - 2)^2 \cos \delta_{cp}}{256\{1 + \frac{1}{64}\lambda_c^2(\lambda_c^2 - 2)^2(8\sqrt{7}\cos \delta_{cp} - 23)\}}, \quad (14)$$

$$\sin^2 \theta_{23} = \frac{112 - \lambda_c^2\{224 - 424\lambda_c^2 - 32\sqrt{7}(\lambda_c^2 - 2)^2 \cos \delta_{cp}\}}{256\{1 + \frac{1}{64}\lambda_c^2(\lambda_c^2 - 2)^2(8\sqrt{7}\cos \delta_{cp} - 23)\}}. \quad (15)$$

In the Ref [20], three data of 1σ ranges are specified regarding $\sin^2 \theta_{23}$. They are 0.400-0.461 and 0.573-0.635 (N.H) and 0.569 - 0.626 (I.H). From Eq.(15), with the limit, $0 \leq |\cos \delta_{cp}| \leq 1$, we get the bound of $\sin^2 \theta_{23}$ as 0.427 - 0.463 and hence out of all three possible 1σ bounds of $\sin^2 \theta_{23}$, two are strongly ruled out and our analysis is very well fitted with the first one [fig.4]. This supports the existence of θ_{23} to lie within the first octant. It is to be noted that the best fit [20] of $\sin^2 \theta_{23}$ i.e. 0.427 coincides with our analysis when $\delta_{cp} = 0$.

From Eqs.(13 - 14), this is clear that Dirac phase δ_{cp} affects the prediction of all the three mixing angles which is different from BM case where only θ_{12} is affected by δ_{cp} (Eq. (9)). It seems that the situation is now much more complicated than the BM case. If our initial choice for U_ν as strictly BL and U_l as CKM types were appropriate, then on placing the best fit [20] results at least for two of the three parameters in any two out of the three Eqs. (14-16), the predictions of δ_{cp} from the respective equations must coincide. The situation is as if for one unknown parameter δ_{cp} , there are more than one equations. We first solve Eq.(14) with the best fit value of $\sin^2 \theta_{13}$ [20], to find out $\cos \delta_{cp}$ and do the same for Eq.(15) with $\sin^2 \theta_{12}$. But surprisingly, we find the predictions of $\cos \delta_{cp} \approx 0.70$ (i.e $\delta_{cp} \approx 0.25\pi$) is same from both of the equations. In the next step, we put $\cos \delta_{cp} \approx 0.70$ in Eq. (16), and get $\sin^2 \theta_{23} \approx 0.44$ which is close to best-fit result $\sin^2 \theta_{23} = 0.427$ [20]. These analyses are illustrated graphically in the figs.2-4.

With strict BL mixing as the 1st approximation ($U_\nu = U_{BL}(\lambda_c)$) and along with a unitary charged lepton mixing matrix ($U_l(\lambda_c, \delta_{cp})$) of CKM type, the predictions are summarised as follows.

$$\sin^2 \theta_{13} = 0.0245, \quad \sin^2 \theta_{12} = 0.3209, \quad \sin^2 \theta_{23} = 0.4533, \quad \delta_{cp} = 0.2515\pi. \quad (16)$$

From Eq. (13), we work out the CP violation Jarskog invariant parameter as $J_{cp}^{BL} = \text{Im}[U_{e1}^* U_{\mu 1}^* U_{e3} U_{\mu 1}]$,

$$|J_{CP}^{BL}| = \frac{9\sqrt{7}}{4096}\lambda_c\{28 - 8\lambda_c(1 + 8\lambda_c - \lambda_c^2) + 57\lambda_c^4\} \sin \delta_{cp} \approx 0.0304 \sin \delta_{cp}. \quad (17)$$

If we choose $\delta_{cp} \approx 0.2515\pi$, as per as the prediction, then we get $J_{CP}^{BL} \approx 0.0216$.

4 Prediction of effective electron neutrino mass m_{ee} in $0\nu\beta\beta$ decay.

The effective electron neutrino mass m_{ee} appeared in neutrinoless double decay ($0\nu\beta\beta$) is given as

$$m_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2| \quad (18)$$

where m_i 's are the masses of the three neutrino mass eigenstates. Using Eq.(12), with $\lambda_c = 0.2257$, and $\delta_{cp} \approx 0.2515\pi$ we get

$$m_{ee} = |0.5262m_1 + 0.4056m_2 + 0.06954m_3 + (0.1953m_1 - 0.1314m_2 - 0.0640m_3) \cos \delta_{cp}|. \quad (19)$$

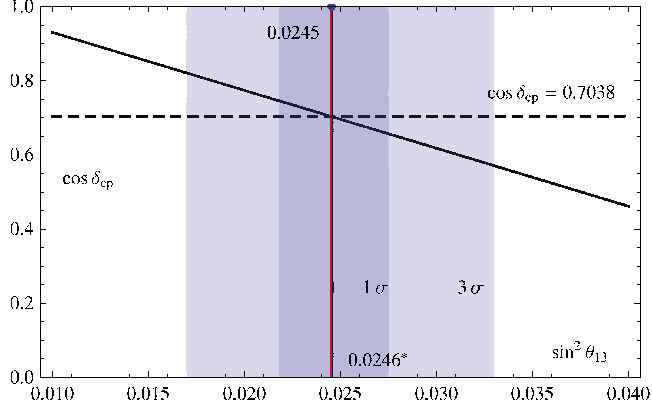


Figure 2: The dependence of $\cos \delta_{cp}$ on $\sin^2 \theta_{13}$ for BL case with charged lepton correction.

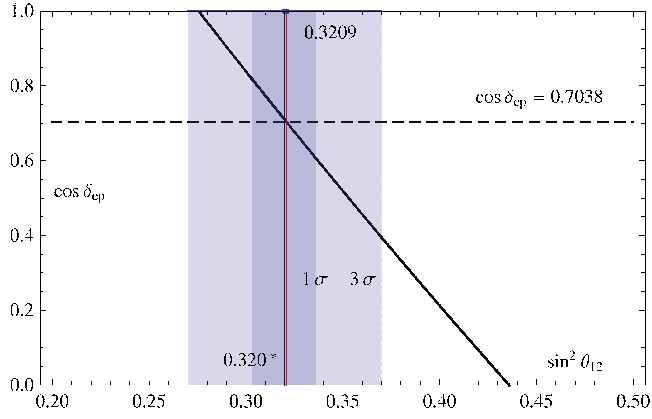


Figure 3: The dependence of $\cos \delta_{cp}$ on $\sin^2 \theta_{12}$ for BL case with charged lepton correction.

For N.H case with m_1 as the smallest mass, we have,

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}. \quad (20)$$

We impose the Cosmological upper bound for $\Sigma m_i \leq 0.28 eV$ [21] in our analysis. We fix $\Delta m_{21}^2 \sim 7.62 \times 10^{-5} eV^2$ (best-fit) [20] and $\Delta m_{31}^2 \sim 2.55 \times 10^{-3} eV^2$ (best-fit) [20] and plot Σm_i taking lowest mass m_1 as free parameter and get the quasidegenerate upper limit for m_1 as 0.088 eV (fig. 6). We then plot m_{ee} with respect to m_1 for three different cases concerning Majorana phases: $(+m_2, +m_3)$, $(-m_2, +m_3)$ and $(+m_2, -m_3)$ (fig.7). Concerning this three cases the predictions for m_{ee} under the quasidegenerate limit of $m_1 \sim 0.088 eV$ are as follows.

$$\begin{aligned} (+m_2, +m_3) &: 0.0045 eV \leq m_{ee} \leq 0.0891 eV, \\ (-m_2, +m_3) &: 0 \leq m_{ee} \leq 0.0335 eV, \\ (+m_2, -m_3) &: 0.0023 eV \leq m_{ee} \leq 0.0839 eV, \end{aligned} \quad (21)$$

where \pm signs before $m_{2,3}$ indicate the Majorana CP phases. Pascoli and Petcov[22] showed that if the neutrino mass ordering were of normal type, then $|m_{ee}|$ would satisfy $0.001 eV \leq |m_{ee}|$ which is consistent with the cases discussed above except $(-m_2, +m_3)$. There is an upper bound of neutrino mass parameter $m_{ee} \leq 0.27 eV$ [23] which appears in the neutrinoless double beta decay experiments. The upper bounds of m_{ee} for the three cases under quasidegenerate limit of m_1 satisfy this condition.

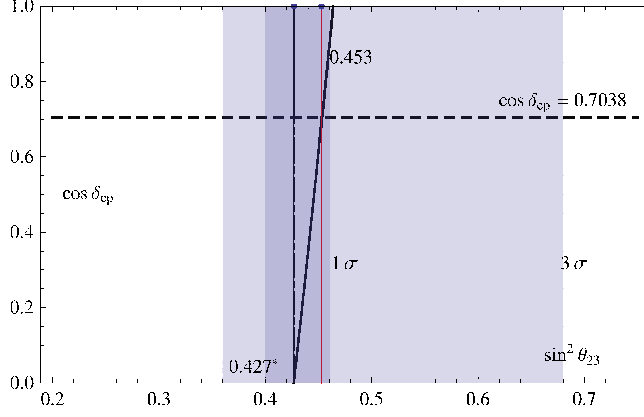


Figure 4: The dependence of $\cos \delta_{cp}$ on $\sin^2 \theta_{23}$ for BL case with charged lepton correction.

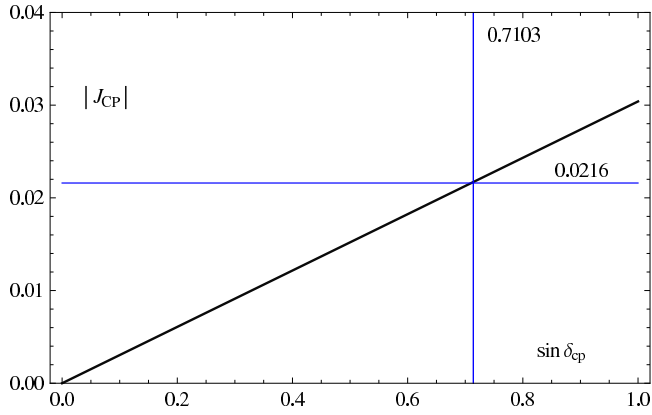


Figure 5: The variation of J_{cp} with $\sin \delta_{cp}$ for BL case with charged lepton correction.

5 Summary

We have discussed the shortcomings of BM model where after considering the charged lepton correction, we are unable to lower the solar angle below $\sin^2 \theta_{12} = 0.3407$ (i.e., $\tan^2 \theta_{12} = 0.52$), although the prediction of θ_{13} and θ_{23} comply with the experimental results. Boucenna *et.al* has introduced a new mixing pattern called Bi-Large mixing where Cabibbo angle (λ_c) seeds the whole parametrization [Eqs.(1)-(3)]. We assume U_l to be of CKM type [Eq.(5)] and construct U_{PMNS} . This new model although phenomenological, is characterized by the following significant features: **(a)** Any other possibilities than θ_{23} to lie within the first octant, are sharply ruled out, **(b)** The predictions $\sin^2 \theta_{13} \sim 0.0245$ and $\sin^2 \theta_{12} \sim 0.3209$ are in precise agreement with the experimental best-fit values. We obtain $\sin^2 \theta_{23} \sim 0.453$ which is close to the best-fit value (within 1σ range), and **(c)** $\delta_{cp} \sim 0.2515\pi$ and $|J_{CP}^{BL}| = 0.0304 \sin \delta_{cp} \sim 0.0216$. The same U_l (CKM type), when incorporated with U_{BM} was partly successful in complying with the experimental results because there it imposes a condition of $\delta_{cp} \rightarrow 0$, (i.e there is no CP violation) in order to lower the solar angle. Whereas this shortcoming is removed very easily when we associate the same CKM type U_l with strict BL scheme. Hence the BL mixing scheme is very significant in the light of present experimental results. The model is further strengthened by the fact that the predictions of θ_{13} , θ_{12} and θ_{23} individually depend upon δ_{cp} , without any contradiction. All the three angles agree to the desired results for a single choice of $\delta_{cp} \sim 0.2515\pi$. Finally the model is employed to study the upper bounds of m_{ee} in quasidegenerate limit for three different Majorana CP phases of normal hierarchy. A formal derivation of BL mixing matrix from discrete symmetry is an important aspect for our future investigation.

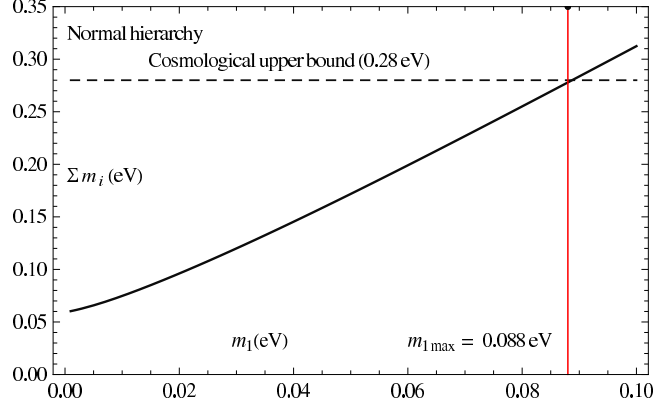


Figure 6: The variation of Σm_i with m_1 . The cosmological upper bound : $\Sigma m_i \leq 0.28\text{ eV}$. The Q.D limit of m_1 is 0.088 eV .

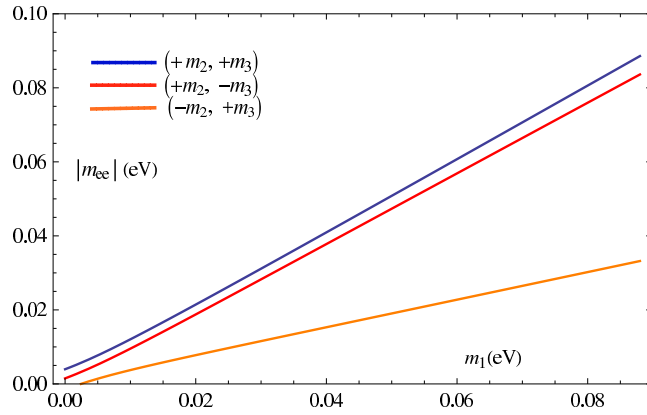


Figure 7: The variation of m_{ee} with m_1 for $(+m_2, +m_3)$, $(-m_2, +m_3)$ and $(+m_2, -m_3)$ CP cases.

Acknowledgement

One of the authors (SR) wishes to convey his heartiest gratitude to Chandan Duarah of Department of Physics, Gauhati University for usefull discussion.

References

- [1] DOUBLE-CHOOZ Collaboration, Y.Abe et al., Phys.Rev.Lett.**108**,131801 (2012),[arXiv:1112.6353](#) [hep-ex].
- [2] DAYA-BAY Collaboration,F.An et al.,Phys.Rev.Lett.**108**,171803 (2012), [arXiv:1203.1669](#)[hep-ex].
- [3] RENO Collaboration, J. Ahn et al.,Phys.Rev.Lett.**108**,191802 (2012),[arXiv: 1204.0626](#)[hep-ex].
- [4] T2K Collaboration, K. Abe et al., Phys.Rev.Lett.**107**, 041801 (2011), [arXiv: 1106.2822](#)[hep-ex]
- [5] R. Nichol, Plenary talk at the Neutrino 2012 conference, [http: //neu2012.kek.jp/](http://neu2012.kek.jp/).
- [6] P.F Harrison, D.H Perkins and W.G Scott, Phys.Lett.**B530** 167 (2002), [hep-ph/0202074](#)[hep-ph].
- [7] G. Altarelli, F. Feruglio, Phys. Rep.**320**, 295 (1999)
- [8] V. D. Barger, S.Pakvasa, T. J. Weiler, K. Whisnant, Phys. Lett.**B437**, 107 (1998)
- [9] N. Nimai Singh and M. Patgiri, Intl. J. Mod. Phys.**A17**,3629 (2002).
- [10] K.S. Babu, E. Ma and J.W.F Valle, Phys. Lett.**B552**, 207 (2003), [hep-ph/0206292](#).
- [11] E. Ma, Phys.Rev.**D66** 117301 (2002),[hep-ph/0207352](#).

- [12] G. Altarelli and F. Feruglio, Nucl. Phys.**B720**, 64 (2005),**hep-ph/ 0504165**.
- [13] G. Altarelli and F. Feruglio, Nucl.Phys.**B741** (2006),**hep-ph/0512103**.
- [14] G. Altarelli and F. Feruglio, **hep-ph/0306265**.
- [15] S. M. Boucenna, S. Morisi, M. Tortola and J. W. F.Valle, Phys.Rev. **D86**,051301(2012), **arXiv:1206.2555**[hep-ph]
- [16] G. Fogli et al., Phys.Rev.**D86**,(2012) 013012, **arXiv:1205.5254**.
- [17] M. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP **1004**, 056 (2010), **arXiv: 1001.4524**.
- [18] C. Duarah, A. Das, N. Nimai Singh, Phys.Lett. **B718** (2012) 147-152, **arXiv:1207.5225**.
- [19] H. Fritzsch, Z-zhong Xing, Phys.Rev.**D57**, (1998) 594-597, **arXiv:hep-ph/9708366**.
- [20] D.V. Forero, M. Tortola, J.W.F Valle, Phys.Rev.**D86**(2012) 073012, **arXiv: 1205.4018v4**.
- [21] S.A.Thomas, F.B. Abdalla, and Ofer Lahav, Phys.Rev.Lett **105**, 031301 (2010).
- [22] S. Pascoli, S.T.Petcov, Phys. Rev **D77**,1130039 (2008).
- [23] Ng.K. Francis, N. Nimai Singh, J.Mod.Phys.**2** (2011) 1280-1284, **arXiv:1206.3434**; Nucl. Phys. **B86** (2012) 19-32.